ІННОВАТИКА В ПРИРОДНИЧО-НАУКОВІЙ ОСВІТІ: ПРОЄКТИ, ПРОГРАМИ, МЕТОДИКИ, ТЕХНОЛОГІЇ

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INTEGRATING DIFFERENTIAL EQUATIONS INTO HIGHER EDUCATION STEM: PROJECT-BASED APPROACH AND DIGITAL TECHNOLOGIES

Abstract. The article addresses the actual challenges in higher education in physics and mathematics and substantiates the feasibility of its modernization through the integration of the differential equations course within the framework of STEM education. Particular attention is paid to bridging the gap between the theoretical training of higher education students and the practical demands of modern science and engineering practice. STEM-oriented approach to teaching differential equations is proposed, based on a combination of problem-based learning and project-based learning. This approach facilitates a transformation of the educational process in which students act as active researchers, while instructors assume the role of facilitators and coordinators of learning and research activities. The practical implementation of the proposed approach involves a step-by-step project workflow aimed at solving applied problems in mathematical modelling. To illustrate the applied potential of differential equations, a series of case studies is presented, covering problems related to thermal, acoustic, hydrodynamic, electromagnetic, and nonlinear oscillatory processes. The role of specialized applied software tools in solving such problems is highlighted, and a comparative analysis of widely used platforms, including Wolfram Mathematica, MATLAB, and Python, is conducted. The implementation of a STEM-oriented approach in teaching differential equations enhances students' motivation and contributes to the development of key professional competencies necessary for training competitive specialists in the field of natural and mathematical education.

Key words: STEM project, teaching differential equations, digital tools, interdisciplinary approach, higher education students.

Relevance and methodological conditions. The modern educational paradigm is undergoing radical transformations, moving away from the traditional knowledge-based approach that focused on the mechanical memorization of facts and formulas. Instead, the competency-based approach is gaining importance, which aims to develop critical thinking skills, systematic analysis and the ability to solve complex problems in real-life situations in higher education students. These changes are dictated by both global trends and the demands of the labour market, which requires specialists capable of interdisciplinary interaction and flexible response to a rapidly changing world.

Unfortunately, classical courses in higher mathematics, in particular differential equations, in higher education institutions often remain outside these transformations. Like the school curriculum, they are often detached from real life and practical applications, leading to a loss of motivation among higher education students who do not see the practical value of studying abstract concepts. This problem is not merely didactic, but fundamental and systemic: the gap between theoretical abstraction and the applied world creates a kind of «chasm» in understanding. A student who does not understand how the theory of differential equations can be used to model physical processes and technical problems has no internal incentive to master the material in depth. The introduction of a STEM-oriented approach at the higher education level is not just a «modernization» but a critically important condition for overcoming this gap. This allows differential equations to be transformed from an end in themselves into a powerful tool for describing and understanding various processes in the surrounding world, which radically changes the perception of the subject [1].

Tasks that require the use of mathematical tools in the context of other sciences (physics, economics, programming) teach higher education students not only to perform algorithms, but also to analyse problems from different viewpoints, evaluate the reliability of data, and choose the best strategies for solving them. This shapes an analytical mind-set. Real-world problems often do not have a single solution. The interdisciplinary context of STEM-oriented education encourages students to search for non-standard mathematical models and creatively combine knowledge from different fields to generate innovative solutions. Thus, an interdisciplinary approach to learning is critical for acquiring the necessary skill set that stimulates critical thinking, creativity, and the readiness of higher education seekers for real-world challenges [8].

Conceptual foundations of STEM-oriented teaching of differential equations. As is well known, STEM education is based on the integration of science, technology, engineering, and mathematics and serves as a conceptual basis for the modernization of higher mathematical education. Its key principles – integration and practical orientation – demonstrate the connection between science and life, and contribute to the development of critical and systematic thinking, teamwork skills, and the ability to solve complex problems of today.

The central methodologies for implementing these principles are Problem-Based Learning (PBL) and Project-Based Learning (PjBL).

PjBL is an educational technology that involves active research into real-world challenges and problems. PjBL focuses on the engineering design process, where higher education students identify a problem, conduct research, develop solutions and, if necessary, create a prototype. PBL, in turn, uses a problem as a starting point for acquiring knowledge. Both methods, being multidisciplinary in nature, significantly increase student motivation by engaging them in solving real physical, economic, and technical problems.

PBL and PjBL are not just a set of techniques, but a systematic approach that transforms the role of the teacher from a «source of knowledge» to a «leader of educational and scientific research». Students cease to be passive listeners and take responsibility for their own learning. This transition from passive consumption of information to its active creation in the form of specific research is a fundamental change in the entire educational process, which directly correlates with the requirements of STEM education and the labour market. Thanks to this approach, the learning process mimics real scientific research or engineering work, helping higher education seekers develop key competencies such as teamwork, critical thinking, and readiness to solve real-world problems [2, 5].

Development and implementation of a comprehensive STEM project on differential equations. Project-based learning based on real-world models is a highly effective method of teaching differential equations, as it allows students to move from an exclusively algorithmic style of learning to a deep understanding of the content and practical significance of the material [3, 7]. This approach can be implemented using a step-by-step plan that combines scientific and engineering methods:

1. Setting the task/question. Students formulate a real-world problem, such as «How to predict bridge vibrations during gusts of wind?» or «How does heat transfer occur in an object?».
2. Background research. At this stage, learners search for information about existing mathematical models and solutions using scientific sources and online resources.
3. Formulating a hypothesis/requirements. Students make assumptions about the expected behaviour of the model or compile a list of requirements that their solution must meet.
4. Building and solving the model. This is the main stage, where theoretical knowledge from the course is applied in practice to build a differential model and its analytical or numerical solution.
5. Testing and data analysis. The results obtained are evaluated and compared with real data. Students analyse and interpret the conclusions, filtering out false or irrelevant solutions.
6. Presentation. The final stage, where the project results are demonstrated and their practical value is discussed.

In modern STEM education, differential equations are a fundamental tool for describing and analysing processes that change over time and space. For future specialists in various fields of training, particularly technical ones, mastery of mathematical modelling methods is a key competence, as it forms the basis for solving most engineering problems [6]. From heat transfer in materials and vibrations in mechanical systems to controlling electronic circuits or predicting the behaviour of complex technical objects, all these processes can be effectively formulated and investigated using differential equations.

In the educational process, the use of case studies based on real or near-real engineering situations makes it possible to demonstrate the practical significance of mathematical methods, increase the motivation of students, and ensure integration between mathematics, physics, computer science, technology, and engineering. Such cases promote the development of critical thinking skills, data handling, the application of numerical methods, model analysis, and engineering decision-making.

The cases below illustrate how differential equations help model and study various processes. They demonstrate that mathematical modelling is not an abstract theory, but an effective tool in engineering practice and an important component in the training of competent STEM professionals.

Let us consider some examples of projects involving the construction of mathematical models for the study of dynamic processes in various fields.

<u>Case 1.</u> *Thermal dynamics of a building's volumetric-spatial system (energy efficiency model). Suitable for predicting temperatures in rooms/buildings, assessing heat loss, optimising insulation, and analysing responses to the switching on/off of heating or solar heating.*

Assumptions/preserved: let the materials be linear (thermal conductivity does not depend on *T*), heterogeneous parameters in space are possible, convection at the outer edges is linear, then let us consider the task.

Mathematical model (PDE, 3D)

$$pc_p \frac{\partial T(x,t)}{\partial t} = \nabla * \left(k(x)\nabla T(x,t)\right) + q(x,t).$$

Boundary conditions (examples): on the outer surface (convection + radiation)

$$-k\frac{\partial T}{\partial n} = h\left(T - T_{out}(t)\right) + \sigma\varepsilon\left(T^4 - T_{sky}^4\right),$$

at internal joints – continuity of *T* and heat flow.

Initial conditions

$$T(x,0) = T_0(x).$$

The following solution methods can be applied: modal decomposition for simple geometries; finite element method (FEM) or finite difference method (FDM) for 2D/3D; reduced-order model (0-D or 1-D) through spatial mode approximation.

The practical significance lies in the fact that we obtain the field *T(x,t)* of cooling/heating time, heat flows through the walls, the effect of improved insulation; it allows us to quantitatively estimate energy savings when changing *K* or *h*.

Expected result: temperature/time maps before equilibrium is established, time series of average temperatures in the room, sensitivity of heat loss to parameters (sensitive analyses), efficiency of energy-saving measures in per cent (%).

Case 2. *Wave modelling and modal analysis of room acoustics (stationary and temporal behaviour). Suitable for determining room resonance frequencies, calculating standing waves, designing sound absorption, and localising sound sources.*

Assumptions: linear acoustics, negligible air flow (small M), adequate boundary conditions (absorbing/rigid walls) can be applied.

Mathematical model (wave equation, 3D)

$$\frac{\partial^2 p}{\partial t^2} = c^2 \Delta p(x,t) + s(x,t),$$

where p is acoustic pressure, c is the speed of sound, and s is the source.

Stationary (harmonic) form at

$$p(x,t) = R(\tilde{p}(x)e^{i\omega t}), \Delta\tilde{p} + k^2 = -\tilde{s}(x), \; k = \frac{\omega}{c}.$$

Boundary conditions:

rigid wall $\frac{\partial \tilde{p}}{\partial n} = 0$,

absorbing (impedance) $\frac{\partial \tilde{p}}{\partial n} + Z\tilde{p} = 0$.

Methods: modal analysis (natural frequencies and natural modes), Helmholtz solution (FEM), absorbing boundary conditions (PML) methods for open spaces; time integrators for non-stationary problems.

Practical significance: finding resonance frequencies and mode shapes ⇒ avoiding standing waves in concert halls/classrooms; quantitative selection of acoustic panels (mode attenuation assessment).

Expected result: natural frequencies and modal shapes, sound level distribution at a given source; estimation of average SPL (sound pressure level) and level reduction after adding absorbers.

Case 3. *Hydrodynamics of pipes (practical model for heating and ventilation systems). Suitable for calculating pressure losses in pipelines, predicting transition to turbulence, and determining the efficiency of heat exchangers and ventilation systems.*

Assumptions: incompressible fluid (for low speeds), stable viscosity; use a thin boundary layer if necessary.

Mathematical model (Navier–Stokes)

$$l\rho\left(\frac{\partial v}{\partial t} + (v * \nabla)v\right) = -\nabla p + \mu\Delta v + f, \quad \nabla * v = 0.$$

Example of reduction – single-axis, stationary, one-dimensional.

Equation of flow in a pipe $\frac{\partial p}{\partial x} = -\frac{f_D \rho v^2}{2D}$, where f_D is the friction coefficient (Re function).

Methods: direct numerical simulation (DNS) for research (expensive); RANS + FEM for engineering estimates; linear stability analysis for critical Re; numerical methods: FVM/FEM.

Practical significance: pressure loss prediction, optimization of diameters/speeds to minimize fan energy consumption, evaluation of filter/air duct efficiency, determination of conditions where turbulence begins (increase in losses).

Expected result: velocity and pressure distributions; pipeline loss values as a function of velocity; determination of mode (laminar/turbulent); recommendations for energy optimization (reduction of Re or redistribution of velocities).

Case 4. *Electromagnetic waves in waveguides/signal lines – a model for EMC and energy transmission. Suitable for analysing electromagnetic wave propagation, determining transmission modes in waveguides, compatibility issues (EMC), losses and impedance matching in power systems/radio electronics.*

Assumptions: linear isotropic media, possible dielectric and conductive losses; for long lines – line transmission model.

Mathematical model (Maxwell → wave equation)

$$\nabla \times E = -\frac{\partial B}{\partial t}, \;\; \nabla \times H = J + \frac{\partial D}{\partial t}.$$

Reduction to a wave equation for a field, for example, for E

$$\nabla^2 - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = \mu \frac{\partial J}{\partial t}.$$

Line transmission model (1D)

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t} - RI, \; \frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t} - CV,$$

where *G, R, L, C, G* are parameters per unit length.

Boundary conditions: appropriate at the boundaries (impedance, short circuit, open circuit, or PML to reproduce infinite space).

Methods: modal decomposition in regular waveguides; time-domain methods (FDTD) or FEM in the frequency domain; transmission model solution for short/long lines; *S*-parameters for reflection/transmission estimation.

Practical significance: determination of passbands, losses, reflections; design of matching devices; evaluation of the influence of screens and filters on EMC; minimization of losses in power/signal transmission lines.

Expected result: critical frequencies (modes), field distribution, reflection/transmission coefficients, optimal matching parameters for minimizing reflection and losses.

Case 5. *Nonlinear oscillatory systems and resonances (Duffing, Van der Pol) – analysis of steady-state and transient modes. Suitable for predicting nonlinear responses of structures/electrical circuits/optical resonators at high amplitudes; analysing abrupt transitions (bifurcations), resonance capture, damping, and energy extraction.*

Assumptions: nonlinearities are important (geometric or material), weak dissipation or the presence of external forced excitation.

Classical model (Duffing oscillator)

$$m\ddot{x} + c\dot{x} + kx = ax^3 = Fcos(\omega t),$$

where α is the nonlinearity (stiffness) coefficient.

Alternative models Van der Pol (self-excitation)

$$\ddot{x} - \mu\left(1 - x^2\right)\dot{x} + \omega_0^2 = 0.$$

Analysis: asymptotic approximation methods (averaging method, multiple scales), search for stationary amplitude-frequency characteristics, study of stability and bifurcations (how the number of steady-state solutions changes with parameter variation).

Practical significance: prediction of uncontrolled transition to large oscillations in structures; explanation of sharp changes in amplitude in acoustic/mechanical systems; design of nonlinear dampers/energy traps; application in energy structures (vibration energy dissipation).

Expected results: amplitude-frequency curves (including hysteresis/multilevelness), stability maps, parameter thresholds for bifurcation, predictions of probable transitions to chaos with increasing stimulus, recommendations on parameters to avoid unwanted resonances.

Digital tools as a catalyst and instrument for STEM education. The transition from traditional calculations to computer modelling is not just a simplification, but a strategic step that allows us to focus on conceptual understanding instead of routine calculations. Modern computer algebra systems (Maple, Mathematica, MATLAB) and visualization programs (GeoGebra, GeomED, STELLA) are important tools that simplify the modelling of real processes and allow the visualization of solutions, which significantly improves the quality of learning [4].

It is important to understand that these tools are not interchangeable, but complementary. The choice of tool is determined by the type of task: Wolfram Mathematica is better suited for analytical research of equations (searching for a general solution), while MATLAB or Python is better for modelling with large data sets. This means that teachers should teach higher education students not a «specific program» or skills for working with a particular interface, but the logic of choosing tools depending on the task at hand, which is a critical competence for future specialists. For example, Wolfram Mathematica with its Manipulate function is ideal for interactive visualization, allowing students to quickly see how changing parameters affects the solution (e.g., SIR models), while MATLAB or Python are better suited for working with large real-world data sets, such as data on the spread of infection, which is part of a STEM project (*Table 1*).

Table 1

Comparative characteristics of digital tools for teaching differential equations

Criterion	*Wolfram Mathematica*	*MATLAB*	*Python*
Advantages	Exceptional capabilities for symbolic computation, manipulation, and academic research	Primarily focused on numerical calculations, matrix operations, and engineering tasks	Versatility, use of libraries for numerical (NumPy, SciPy) and symbolic (SymPy, Wolfram Client Library) computations
Interface	Notebook format that combines code, output, and documentation in a single file; unified IDE	Excellent integrated development environment (IDE) for debugging and file management	Flexible environments (IDE, Jupyter Notebooks) that allow you to adapt your workflow to specific needs
Visualization capabilities	Powerful graphics capabilities; interactive visualization features such as Manipulate	Strengths in graphing and data visualization	Extensive visualization capabilities thanks to matplotlib, Plotly, and other libraries
Scope of application	Academic, pure mathematics, theoretical physics	Engineering, signal processing, numerical methods, industry	Data, machine learning, scientific research, web development; universal application
Accessibility	Commercial license	Commercial license	Open source

Methodological integration: from theory to practice. For the successful implementation of the project approach, several key methodological recommendations must be followed. First of all, it is important to organize the work. The problems for the projects should be complex enough to encourage students to form interdisciplinary groups for collaboration. This will allow them to effectively distribute roles and apply different skills to achieve a common goal [9].

The role of the teacher changes from that of a mentor who imparts knowledge to that of a facilitator who guides students' thinking. The teacher does not provide ready-made solutions, but helps students identify the problem, asks leading questions and provides timely feedback.

The assessment system must also be adapted to project-based learning. Traditional assessment, based solely on the final result (exam), does not reflect the full scope of the work. Instead, it is proposed to use a comprehensive system that includes intermediate stages (e.g., assessment of background research, hypotheses), assessment of teamwork, and assessment of the final product (report, presentation, model). This will allow for an objective assessment not only of the final result, but also of the learning process itself, which is the main goal of the STEM approach.

Conclusions. The implementation of a STEM-oriented approach to teaching differential equations in higher education institutions, based on project-based learning and the use of digital technologies, is an effective strategy for overcoming the key challenges of modern education. This approach significantly increases students' motivation and interest in the subject, as they see for the first time with their own eyes the practical value of higher mathematics for solving real-world problems.

In addition, project-based learning contributes to the development of a range of key competencies that are critical for success in the labour market, including critical thinking, teamwork skills, initiative and lifelong learning skills. This allows for a deeper conceptual understanding of the material, which surpasses the results obtained using traditional methods.

For further development of this area, it is necessary to develop comprehensive training courses, methodological guides and banks of applied STEM tasks adapted for use in Ukrainian higher education institutions. It is also important to conduct research on the effectiveness of this approach, using both quantitative and qualitative indicators to assess learning outcomes. The concept of STEM education in Ukraine is only just taking shape at the state level, and there is a need for professional training of scientific and pedagogical workers. The introduction of innovative methods in higher education will have a scaling effect on the entire education system, as graduates who will become future teachers, engineers and scientists will transfer these approaches to lower levels of education and to their individual professional environments. Thus, the proposed methodology is not just a local improvement of a single course, but a contribution to the overall transformation of science and mathematics education in Ukraine, which is in line with the main goal of the STEM conference.

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ІНТЕГРАЦІЯ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ У STEM-ОСВІТУ ВИЩОЇ ШКОЛИ: ПРОЄКТНИЙ ПІДХІД І ЦИФРОВІ ТЕХНОЛОГІЇ

Анотація. У статті розглянуто актуальні проблеми вищої фізико-математичної освіти та обґрунтовано доцільність її модернізації шляхом інтеграції курсу диференціальних рівнянь у контекст STEM-освіти. Основну увагу приділено подоланню розриву між теоретичною підготовкою здобувачів вищої освіти та практичними потребами сучасної науки й інженерної діяльності. Запропоновано STEM-орієнтований підхід до викладання диференціальних рівнянь, що базується на поєднанні проблемно-орієнтованого та проєктного навчання. Такий підхід сприяє трансформації освітнього процесу, за якої студент виступає активним дослідником, а викладач – фасилітатором і координатором навчально-дослідницької діяльності. Практична реалізація підходу передбачає поетапний алгоритм роботи над проєктами, спрямованими на розв'язування прикладних задач математичного моделювання. Для ілюстрації прикладного потенціалу диференціальних рівнянь наведено низку кейсів, що охоплюють задачі з теплових, акустичних, гідродинамічних, електромагнітних та нелінійних коливальних процесів. Показано роль спеціалізованих програмних засобів у розв'язуванні таких задач та здійснено порівняльний аналіз можливостей Wolfram Mathematica, MATLAB і Python. Впровадження STEM-орієнтованого підходу у викладання диференціальних рівнянь підвищує мотивацію здобувачів освіти та сприяє формуванню ключових професійних компетентностей, необхідних для підготовки конкурентоспроможних фахівців у галузі природничо-математичної освіти.

Ключові слова: STEM-проєкт, викладання диференціальних рівнянь, цифрові інструменти, міждисциплінарний підхід, здобувачі вищої освіти.

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